

Indian Institute of Technology, Kharagpur

Date: 21/09/2011 Time: 45 mins Full Marks: 10
Class Test (Autumn) Semester 2011-12 No. of Students: 65 Sub. No. MA 21007
Subject Name: Design and Analysis of Algorithms

Instruction: Answer all questions.

Question 1 [1+1+1=3 marks]

- a) Introduce statements to increment count at all appropriate points in the following recursive procedure for fast computation of integer exponents. Find the exact value of count when the procedure terminates (assume that the initial value of count is 0).

```
procedure EXPONENT(x, i)
  if (i = 0) then return 1;
  else if (i is even) then return EXPONENT(x, i/2)^2;
  else return x * EXPONENT(x, (i - 1)/2)^2;
  end if
end EXPONENT
```

- b) Explain why this algorithm uses $O(\log(i))$ time, under standard unit-cost assumption that integers can be multiplied in $O(1)$ time.
- c) Explain why this algorithm uses $O(\log(i))$ space, under standard unit-cost assumption that integers consume $O(1)$ space.

Question 2 [$\frac{1}{2} + 1 + \frac{1}{2} + 2 = 4$ marks]

- a) Show that the number of comparisons required to sort n elements using Merge sort is given by:

$$T(1) = 0$$
$$T(n) = T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + T\left(\left\lceil \frac{n}{2} \right\rceil\right) + n - 1$$

where $\lfloor x \rfloor$ denotes the integer part of x and $\lceil x \rceil$ denotes the smallest integer $\geq x$.

- b) Show that the solution to the above recurrence is: $T(n) = n \lceil \log n \rceil - 2^{\lceil \log n \rceil} + 1$
- c) Is Heapsort stable? Justify your answer.
- d) Consider the following array for sorting (sorting from smallest to largest)

13 19 9 5 12 8 7 4 11 2 6 21

- (i) Draw the array after the FIRST iteration of the large loop in a Selection sort.
- (ii) Draw the array after the FIRST iteration of the large loop in an Insertion sort.
- (iii) Draw the array after TWO recursive calls of Mergesort are completed, and before the final merge step has occurred.
- (iv) Draw the array after applying Quicksort's partition function and using 13 for the pivot. Clearly describe the working of the partition function with this example.

Question 3 [1 + 2 = 3 marks]

a) Starting with an empty binary search tree, insert the following items (in this order):

31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46

b) Let T be the binary search tree obtained in part (a) above.

(i) Draw the binary search tree T_1 after performing DELETE(46), SPLAY(45) on T .

(ii) Draw the binary search tree T_2 after performing DELETE(45), DELETE(44), SPLAY(43) on T_1 .

(Here DELETE(x) deletes node with key x and SPLAY(x) performs a sequence of rotations to make the node with key x become the root of the binary search tree.)

—The End—

9 8
7 6

9 5 1 2 8 2
6 5 4 7 1 9

